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13MCA12

First Semester MCA Degree Examination, Dec.2013/Jan. 2014
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Prove that $[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)] \leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)]$ is a tautology. **(07 Marks)**

b. Negate each of the following and simplify the resetting statement :
i) $p \rightarrow (\neg q \wedge r)$ ii) $(p \vee q) \wedge \neg(\neg p \wedge q)$ iii) $q \rightarrow \neg[(p \vee q) \wedge r]$. **(07 Marks)**

c. Write the following argument in symbolic form and establish the validity
If Rochelle gets supervisor's position and works hard, then she gets a raise, if she gets the raise, then she will buy a new car. She has not purchased a new car. Therefore either Rochelle did not get the supervisor's position or she did not work hard. **(06 Marks)**

2 a. For the universe of all integers, let p(x), q(x), r(x), s(x), t(x) be the following open statements :
i) p(x) : x > 0 ii) q(x) : x is even iii) r(x) : x is perfect square iv) s(x) : x is divided by 4 v) t(x) : x is divided by 5
Write the following statements in symbolic form and determine whether they are true or false. **(06 Marks)**
i) If x is even then x is not divisible by 5
ii) No even integer is divisible by 5
iii) If x is even and perfect square, then x is divisible by 4
iv) If x is perfect square then it is positive.

b. Establish the validity of the following argument with reasons
$\forall x\,[p(x) \rightarrow q(x) \wedge r(x)]$
$\forall x[p(x) \wedge s(x)]$
$\therefore \forall x[r(x) \wedge s(x)]$.

c. Prove that for every integer n, n^2 even if and only if n is even. **(06 Marks)**

d. Identify the bound variables and free variables in each of the following statements : **(06 Marks)**
i) $\forall y\, \exists z\, [\cos(x + y) = \sin(z - x)]$
ii) $\exists x\, \exists y\, [x^2 - y^2 = z]$.

3 a. Prove that two sets S and T are disjoint if and only if $S \cup T = S \Delta T$. **(02 Marks)**

b. Find $\overline{A \Delta B}$, for sets A and B. **(06 Marks)**

c. How many permutations of the 26 letters of the alphabet contain **(06 Marks)**
i) either the pattern "OUT" or pattern "DIG"
ii) neither the pattern "MAN" nor the pattern "ANT".

d. In a coffee shop there six kinds of muffins eight kinds of sandwiches and five beverages (two hot and three cold). Find the number of ways in which a person can have either a muffin and a hot beverage or a sandwich and cold beverage? **(05 Marks)**

4 a. Prove that for all $n \in z^f$, $n > 3 \Rightarrow 2^n < n!$. **(03 Marks)**

b. Define Fibonacci numbers recursively. If F_i, i = 0, 1, 2 . . . . are Fibonacci numbers, prove that $\sum_{i=0}^{n} F_i^2 = F_n \times F_{n+1}, \forall n \in 2^+$. **(04 Marks)**

c. Find the greatest common divisor of 1369 and 2597. Express it as linear combination of these numbers. **(06 Marks)**

d. A bank pays 6% (annual) interest on savings, compounding the interest monthly. If one deposits Rs. 1000 on the first day of May then how much will this deposit be worth a year later. **(04 Marks)**

5 a. For each of the following functions, determine whether it is one-to- one and into. If yes, find inverse of it.

i) $f: Z \to Z$, $f(x) = 2x + 1$

ii) $f: Q \to Q$, $f(x) = 2x + 1$

iii) $f: R \to R$, $f(x) = x^2 + x$. **(06 Marks)**

b. Let m, n be positive integers with $1 < n \leq m$. Then prove that $S(m + 1, n) = S(m, n - 1) + nS(m, n)$, where $S(m, n)$ is the stirling number of 2^{nd} kind. **(05 Marks)**

c. Let $A = B = R$. determine $\pi_A(D)$ and $\pi_B(D)$ for the following set, $D \subseteq A \times B \cdot D = \{(x, y) \mid x = y^2\}$. **(04 Marks)**

d. Let ABC be equilateral with AB = 1 cm. Show that if we select to points in the interior of the triangle, there must be at least two points whose distance apart is less than 1/3 cm. **(05 Marks)**

6 a. On the set A of all lines in R^2, define the relation R for two lines ℓ_1, ℓ_2 by $\ell_1 R \ell_2$ if ℓ_1 is perpendicular to ℓ_2. Is R reflexive, symmetric, antisymmetric or transitive? **(04 Marks)**

b. For A = {a, b, c, d, e}, the Hasse diagram for the poset (A, R) is shown in Fig. Q6(b).

i) Determine the relation matrix for R

ii) construct the digraph associated with R

iii) topologically sort the poset (A, R). **(06 Marks)**

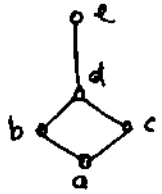

Fig. Q6(b)

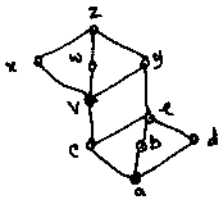

Fig. Q6(d)

c. Let R be an equivalence relation on A. Then prove that

i) $x \in [x]$ ii) $x R y$ if and only if $[x] = [y]$ iii) $[x] = [y]$ or $[x] \cap [y] = \phi$. **(06 Marks)**

d. Define a lattice. Consider the poset (A, R) whose Hasse diagram is given in Fig. Q6(d). Is it a lattice? Justify. **(04 Marks)**

7 a. If G has 25 edges and $\overline{G}$ has 20 edges, how many vertices does G have? **(04 Marks)**

b. If a graph has an Eulerian circuit, then there are no vertices in G with odd degree. **(06 Marks)**

c. Prove that two graphs G and H are isomorphic if and only if $\overline{G}$ and $\overline{H}$ are isomorphic. **(05 Marks)**

d. Define chromatic number. Prove that chromatic number of any bipartite graph with atleast two vertices is 2. **(05 Marks)**

8 a. Find the shortest path from a to e in the following graph (Fig Q8(a)), using Dijkstra's algorithm. **(06 Marks)**

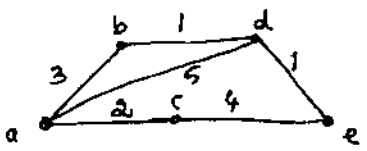

Fig. Q8(a)

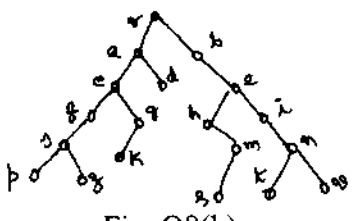

Fig. Q8(b)